Swing Speed vs. Bat and Batter Mass

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In *The Physics of Baseball*, 3rd Edition, Robert K. Adair argues that a plausible model for the relationship between bat speed v and bat mass m can be derived by assuming that the batter puts a fixed amount of energy E into the kinetic energy of the bat plus some fraction of the batter's mass M. Accordingly he writes the formula

$$v = \sqrt{\frac{2E}{m + \epsilon^2 M}}, \tag{1}$$

where ϵ^2 represents the fraction of the batter's mass that shares the kinetic energy with the bat. He notes that observations show that, roughly speaking, the kinetic energy going into the bat and batter are about equal, thereby placing sensible bounds on the value of ϵ^2. In the book he proposes $\epsilon^2 = 1/81$. With this value and for a 162-lb batter swinging a 2-lb (32 oz) bat, the bat has half of the available kinetic energy. For a 200-lb batter swinging a 2-lb bat, the bat has only about 45% of the kinetic energy. Adair goes on to suggest that the energy E provided by the batter is proportional to the mass M of the batter, so that a new relationship can be written that explicitly shows the dependence of v on both m and M:

$$v = k\sqrt{\frac{M}{m + \epsilon^2 M}}, \tag{2}$$

where k is a normalizing constant having the dimensions of velocity.

There is an alternate way to characterize the dependence of bat speed on m and M given by the formula

$$v = k\left(\frac{M}{m}\right)^n, \tag{3}$$

where k is again a normalizing constant with units of velocity. The exponent n is unknown from any first principles but can be determined from experiments.

The best experiment that I know of is the batting cage study of Crisco and Greenwald in which high-speed motion capture cameras were used to track both the baseball and several points along the bat throughout the swing and subsequent collision. From their analysis, they are able to determine the speed of the bat at the point of impact just prior to the collision. The study utilized college and semiprofessional batters, with bat weights in the range 28-31 oz. A summary of their swing speed data is shown in Fig. 1, where the angular velocity of the bat is plotted versus the moment of inertia (MOI) of the bat, both about the knob.[1] The curve is a least-squares fit to the data using a modified version of Eq. 3,[2] with the result n=0.28±0.04.

It is very clear that Eq. 3 cannot be an accurate representation of the dependence of v on m for arbitrary m, since it clearly diverges for small m, unlike Adair's expression. However, I argue that Eqs. 2 and 3 are equivalent over some range of m. In the present context, equivalent means that, given the experimental value of n, there is some choice of ϵ^2 such that $(m/v)dv/dm$ is numerically the same for the two expressions. It is straightforward to derive the necessary expression:

$$\epsilon^2 = \frac{m}{M}\left(\frac{1}{2n} - 1\right) . \qquad (4)$$

Lacking any information about the batters used in the study, I simply assume M=190 lbs. With m=30 oz and $n = 0.28$, I find $\epsilon^2 = 1/129$, a value close to but somewhat smaller than the value 1/81 proposed by Adair. Indeed, one can take the experimental result as an essential confirmation of Adair's value. A value 1/81 would imply n=0.22, which is statistically consistent with the swing-speed data.

If we take seriously the smaller experimental value of ϵ^2, a potentially important consequence is that there is a larger dependence of bat speed on the batter's mass M than would be obtained with the smaller value, as can be seen from Eq. 2 and the following numerical example. Suppose a 200-lb batter swings a 32-oz bat with speed v. By what fraction will the swing speed increase if the batter is 10% heavier (220 lbs) but swings the same bat? For ϵ^2=1/81, v increases by 2.1%; for ϵ^2=1/129, v increases by 2.7%. These number differ by only 0.6%, amounting to less than an 0.5 mph difference in swing speed.

[1] The experiments show that the instantaneous rotation axis of the bat just prior to the collision is very close to the knob.

[2] I ignore here the difference between the mass m and the MOI.

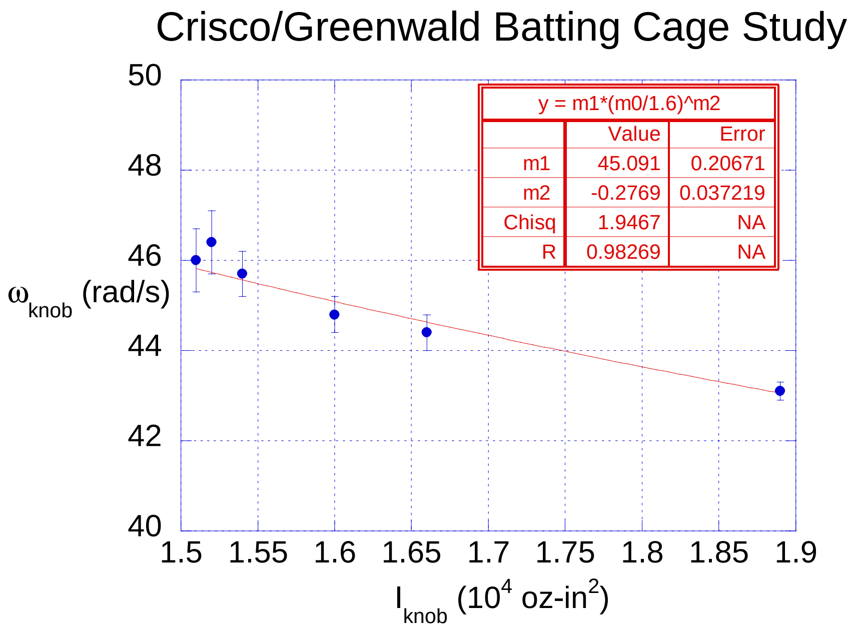

Figure 1: Plot of the angular velocity $\omega_{\rm knob}$ of the bat about the knob just prior to impact versus the $I_{\rm knob}$, the MOI of the bat about the knob. Each point represents the angular velocity of a given bat, averaged over all impacts. The curve is a power-law fit of the form $\omega_{\rm knob} \sim 1/I^n_{\rm knob}$, with the best-fit exponent $n = -0.28 \pm 0.04$